AD-A036 887 NAVAL INTELLIGENCE SUPPORT CENTER WASHINGTON D C TRA--ETC F/G 20/4
GENERATION OF INTERNAL WAVES BY A MOVING REGION OF PRESSURES IN--ETC(U)
FEB 77 S F DOTSENKO, L V CHERKESON

UNCLASSIFIED NISC-TRANS-3897 NL

| OF |

AD
A036887

END

DATE
FILMED
4 — 77

(3) 18

DEPARTMENT OF THE NAVY
NAVAL INTELLIGENCE SUPPORT CENTER
TRANSLATION DIVISION
4301 SUITLAND ROAD
WASHINGTON, D.C. 20390

CLASSIFICATION: UNCLASSIFIED

APPROVED FOR PUBLIC RELEASE: DISTRIBUTION UNLIMITED

TITLE: (6) Generation of Internal Waves by a Moving Region of Pressures in a Sea with a Discontinuity Layer

(Gereratsiya vnutrennikh voln divzhushcheysya oblast'yu davleniy v more so sloyem skachka),

AUTHOR(S): (10) S.F. / Dotsenko, ~~S.F.~~ L.V. / Cherkeson, ~~L.V.~~

PAGES: 12

SOURCE: Morskoy Gidrofizicheskiy Institut, Trudy, No. 3 (59), 1972 Pages 25-38

ORIGINAL LANGUAGE: Russian

TRANSLATOR: C

(14) NISC-TRANS~~LATION NO.~~ -3897

APPROVED P.T.K.

(11) DATE 1 Feb~~ruary 1~~977

(12) 13p.

407 682

bag

GENERATION OF INTERNAL WAVES BY A MOVING REGION OF PRESSURES IN A SEA WITH A DISCONTINUITY LAYER

[Dotsenko, S. F. and L. V. Cherkesov, Generatsiya vnutrennikh voln dvizhushcheysya oblast'yu davleniy v more so sloyem skachka, *Morskoy Gidrofizicheskiy Institut, Trudy*, No. 3 (59), 1972, pp. 25-38; Russian]

Steady internal waves in a sea, caused by displacement of a region of baric disturbances, are investigated. The continuous density stratification is such that the Vaisala-Brunt frequency is zero in the upper and lower layers and positive in the middle layer (discontinuity layer). /25*

It is known from observations that the inhomogeneity of seawater appreciably affects the nature of the wave processes developing in the World Ocean. Thus, whereas in a homogeneous sea the wave processes are manifested most strongly in the surface layer, in a continuously stratified sea, wave disturbances in the deep layers are frequently much more intense than in the surface layer.

Theoretical investigations of wave processes in a continuously stratified sea have been dealt with in a number of studies. Free oscillations were investigated by Fjeldstad,[1] Krauss,[2] Ter-Krikorov,[3,4] Tareyev,[5] Marchuk and Kagan,[6] and others. Internal waves generated by periodically time-varying atmospheric disturbances for density models approximating real density stratifications were discussed in Refs. 7-9. Waves generated by a band of baric disturbances moving at constant velocity perpendicularly to the pressure front, in cases where the density changes over the entire depth or only in the upper layer, were studied in Refs. 10 /2
and 11. The present paper examines an analogous problem for a sea with a discontinuity layer.

1. Let a region of baric disturbances of the form

$$p_0 = af(x+vt) \qquad (v>0), \tag{1.1}$$

move over the surface of a continuously stratified sea of constant depth, where a and v are constants, and $f(x_1)$ is an even function equal to zero when $|x_1| > \ell$.

We will study in a linear formulation the steady internal waves generated by pressures (1.1), assuming that the density in the undisturbed state $\rho_0 = \rho_0(z)$ changes in accordance with the law

$$\rho_0 = \begin{cases} \rho_1, & z_1 \leq z \leq 0 \\ \rho_1 \exp \kappa(z_1 - z), & z_2 \leq z \leq z_1 \\ \rho_1 \exp \kappa(z_1 - z_2), & -H \leq z \leq z_2, \end{cases} \tag{1.2}$$

*Numbers in the right margin indicate pagination in the original text.

where k and ρ_1 are positive constants and $-H \leqslant z_2 \leqslant z_1 \leqslant 0$.

For such a density model, the expressions for the horizontal (u) and vertical (w) velocities of disturbed motion are obtained as in Ref. 11. Finally, when $v_{N+1} < v < v_N$, we obtain the following representations for u and w:

$$u(x,z) = \begin{cases} \sum_{s=1}^{N} u_s(z) \sin m_s x + u_0(x,z), & x > \ell \\ u_0(x,z), & x < -\ell, \end{cases} \tag{1.3}$$

$$w(x,z) = \begin{cases} \sum_{s=1}^{N} w_s(z) \cos m_s x + w_0(x,z), & x > \ell \\ w_0(x,z), & x < -\ell, \end{cases} \tag{1.4}$$

where

$$w_s = 2\sqrt{2\pi}\, \alpha\, m_s F(m_s) \Delta_1(m_s, z)\left[\Delta'(m_s)\right]^{-1}, \quad u_s = -m_s^{-1} w_{sz},$$

$$\Delta = (m \operatorname{ch} m h_1 - g v^{-2} \operatorname{sh} m h_1) æ_{11} + (m \operatorname{sh} m h_1 - g v^{-2} \operatorname{ch} m h_1) æ_{12}, \tag{1.5}$$

/27

$$\Delta_1 = \begin{cases} \Delta_{11}, & z_1 \leqslant z \leqslant 0 \\ \Delta_{12}, & z_2 \leqslant z \leqslant z_1 \\ \Delta_{13}, & -H \leqslant z \leqslant z_2, \end{cases} \tag{1.6}$$

$$\Delta_{11} = æ_{11} \operatorname{sh} m(z - z_1) + æ_{12} \operatorname{ch} m(z - z_1),$$

$$\Delta_{12} = \exp \frac{1}{2}\kappa(z - z_1)\left[æ_{21} \operatorname{sh} \gamma(z - z_2) - æ_{22} \operatorname{ch} \gamma(z - z_2)\right],$$

$$\Delta_{13} = æ_3 \operatorname{sh} m(z + H),$$

$$æ_{11} = \left\{\left[(g\kappa v^{-2} - m^2)\operatorname{th} m h_3 - \frac{1}{2}\kappa m\right] \operatorname{sh} \gamma h_2 - m\gamma \operatorname{ch} \gamma h_2\right\} \operatorname{ch}^{-1} m h_1,$$

$$æ_{12} = æ_{21} \operatorname{sh} \gamma h_2 - æ_{22} \operatorname{sh} \gamma h_2, \quad æ_{21} = m\left(\frac{1}{2} \operatorname{th} m h_3 - m\right) \operatorname{ch}^{-1} m h_1,$$

$$æ_{22} = m\gamma \operatorname{th} m h_3 \operatorname{ch}^{-1} m h_1, \quad æ_3 = -\exp\left(-\frac{1}{2}\kappa h_2\right) m\gamma \operatorname{ch}^{-1} m h_1 \operatorname{ch}^{-1} m h_3,$$

$$\gamma = \sqrt{m^2 - \kappa g v^{-2} + \frac{1}{4}\kappa^2}, \quad h_1 = -z_1, \quad h_2 = z_1 - z_2, \quad h_3 = H + z_2, \quad \alpha = a(\rho_1 v)^{-1};$$

m_s ($s = 1, \ldots, N$) are positive roots of the equation $\Delta(m) = 0$, numbered in decreasing order; v_k ($k = 1, 2, \ldots$) are the values of $v > 0$, numbered in decreasing order, for which $m = 0$ is a pole of the function $m\Delta_1\Delta^{-1}$; $F(m)$ is the Fourier transform of the function $f(x)$; Oxz is the coordinate system with respect to the moving pressure region, the Ox axis coincides with the undisturbed free surface, Oz points vertically upward; u_0 and w_0 are the even and odd functions of x, exponentially approaching zero with increasing distance from the region of application of baric disturbances. In the expressions for u and w when $v > v_1$, terms harmonic in x are absent.

2. We will analyze the dependence of the number of waves N and their lengths on the basic parameters, and also the behavior of the amplitudes u_s and w_s with depth. It is easy to see that such an analysis includes the case of free stationary waves in an inhomogeneous flow moving at constant velocity v.

The number of harmonic waves forming the velocity field of undamped wave motion /28
is determined, as was noted above, by the relationship between v and the values $v = v_n$, for which $m = 0$ is a pole of the function $m\Delta_1\Delta^{-1}$. It follows from expressions (1.5) and (1.6) that $v = v_n$ should be the solutions of the equation

$$\lim_{m \to 0} m^{-2}\Delta(m) = 0. \qquad (2.1)$$

From (1.5) and (2.1), to find v_n, we obtain the transcendental equation

$$\tan \varkappa = \varkappa \frac{1 - \nu(\delta_1 + \delta_2)}{\nu\left[1 + \frac{1}{2}\varepsilon(\delta_1 + \delta_2)\right] - \varepsilon\nu^2\delta_1\delta_2 - \frac{1}{2}\varepsilon}, \qquad (2.2)$$

where $\nu = gh_2 v^{-2}$, $\delta_1 = h_1 h_2^{-1}$, $\delta_2 = h_3 h_2^{-1}$, $\varkappa = \sqrt{\nu\varepsilon - \frac{1}{4}\varepsilon^2}$. v_1 may be found from (2.2) in the form of a series in powers of $\varepsilon \ll 1$. We finally have

$$v_1 = \sqrt{gH}\left(1 - \varepsilon\beta + O(\varepsilon^2)\right), \quad \beta = \frac{1 + 3(\delta_1 + \delta_2)}{12(1 + \delta_1 + \delta_2)^2}. \qquad (2.3)$$

Since wave motion undamped with distance is possible only when $v < v_1$, it follows from (2.3) that it occurs at velocities of the region of baric disturbances somewhat lower but close to the propagation velocity of long waves in a homogeneous liquid ($0 < \beta < 0.1$ when $\delta_{1,2} \geqslant 0$).

Equation (2.2) has a countable set of roots $v = v_n$ such that

$$v_1 > v_2' > \ldots, \quad \lim_{n \to \infty} v_n = 0. \qquad (2.4)$$

In the general case, they can be found only numerically. For a sea with an inhomogeneous upper layer ($h_1 = 0$), approximate expressions for v_n ($n \geqslant 2$) were obtained

in Ref. 11. In the Boussinesq approximation, asymptotic formulas for v_n when $n \to \infty$ were obtained in Ref. 12. Numerical analysis of the roots of Eq. (2.2) showed that v_n $(n \geqslant 2)$ increase with increasing layer thicknesses and parameter ε, which represents the drop in density in the inhomogeneous layer. Moreover, v_n $(n \geqslant 2)$ are directly proportional to $\sqrt{\varepsilon}$ to a high degree of accuracy, and when $\delta_2 \geqslant 4$, are close to the values obtained in the case $h_3 = +\infty$. Table 1 shows v_n $(n = 1\text{-}5)$ for $h_2 = 10^2$ m, $h_3 = 2 \cdot 10^3$ m, $\varepsilon = 10^{-2}$ and three values of h_1 (m). For large h_1 (deep-lying inhomogeneous layer), v_2 may be fairly large and exceed v_3 by a factor of 7-8. Thus, for example, for $h_1 = h_3 = 10^3$ m, $h_2 = 10^2$ m, $\varepsilon = 10^{-2}$ /29
we have $v_2 = 7.05$ m/sec, $v_3 = 0.98$ m/sec.

Table 1

h_1	v_1	v_2	v_3	v_4	v_5
0	143	1,95	0,66	0,40	0,28
20	144	2,32	0,77	0,45	0,31
100	147	3,49	0,91	0,49	0,33

An analysis of the wavelengths $\lambda_n = 2\pi m_n^{-1}$ showed that λ_1 differs little from the wavelength in the homogeneous liquid, and λ_n $(n \geqslant 2)$ depend substantially on the inhomogeneity of the liquid and decrease with increasing thicknesses of the layers, drop in density ε and decreasing v $(v < v_n)$. At the same time, λ_1 is much smaller than λ_n $(n \geqslant 2)$, which are proportional to $1/\sqrt{\varepsilon}$ to a high degree of accuracy. When $\delta_2 \geqslant 10$, λ_n $(n \geqslant 2)$ are weakly dependent on h_3. When $h_1 = 20$ m, $h_2 = 10^2$ m, $H = 2 \cdot 10^3$ m, $\varepsilon = 10^{-2}$, the values of λ_n $(n = 1\text{-}5)$ for four values of v (m/sec) are presented in Table 2.

Table 2

v	λ_1	λ_2	λ_3	λ_4	λ_5
1,6	1,64	691	–	–	–
0,6	0,23	135	245	–	–
0,4	0,10	85	104	225	–
0,3	0,06	62	70	91	265

Let $\varepsilon = 0$ and $v < v_1 = \sqrt{gH}$. In this case, undamped wave motion is generated by exactly one wave, for which $|u_1|$ and $|w_1|$ increase smoothly with increasing z, and reach their maximum values on the free surface. Numerical calculations showed that the difference of u_1, w_1 and λ_1 for $0 \leqslant \varepsilon \leqslant 10^{-2}$ from the corresponding values

when $\varepsilon = 0$ is small and does not exceed 0.1%. Therefore, the shortest wave in the wave group (1.3) may be considered to be an ordinary surface wave slightly perturbed by the inhomogeneity of the sea.

Waves with numbers $n \geqslant 2$ are caused by the vertical change in density. The behavior of $u_n(z)$, $w_n(z)$ $(n \geqslant 2)$ is qualitatively different in the homogeneous and inhomogeneous layers. They have exactly $n - 1$ fields in the interval $z_2 < z < 0$; /3 in the lower layer, the nodal points of amplitudes u_n, w_n are are absent, and their absolute values increase smoothly with increasing z. As is evident from the expression for Δ_{13}, the degree of velocity damping with depth in the lower layer is determined by the ratio $\lambda_n h_3^{-1}$. Furthermore, the law of variation of $w_n(z)$ is quasi-linear, and $u_n(z)$ is almost constant if $\lambda_n h_3^{-1} \gg 1$; when $\lambda_n h_3^{-1} \ll 1$, $u_n(z)$, $w_n(z)$ damp out exponentially with depth. The cases $\lambda_n h_3^{-1} \gg 1$, $\lambda_n h_3^{-1} \ll 1$ are realized when v are close to $v_n (v < v_n)$ and $v \to 0$, respectively.

Similar conclusions may be drawn with regard to the behavior of the amplitudes in the upper homogeneous layer, if the ratio $\lambda_n h_e^{-1}$ is replaced by $\lambda_n h_1^{-1}$. For $h_1 \geqslant 20$ m, $h_2 \geqslant 50$ m, $0 \leqslant h_3 \leqslant 2 \cdot 10^3$ m, $10^{-3} \leqslant \varepsilon_2 \leqslant 10^{-2}$, $|w_n(0)|$ is smaller than $|w_n(z_1)|$ by a factor of over 10^2, and $w_n(z)$ has one nodal point in the upper layer near the free surface; $u_n(z)$ has no nodal points when $z_1 \leqslant z \leqslant 0$. In the inhomogeneous layer, the amplitudes $u_n(z)$, $w_n(z)$ have an oscillatory character in the general case. This is explained by the fact that for internal waves, the wave numbers satisfy the inequality

$$0 < m_s < \sqrt{g\kappa v^{-2} - \frac{1}{4}\kappa^2} \quad (s \geq 2),$$

and therefore the hyperbolic functions in formulas (1.5) and (1.6), which contain the factor γ in their arguments, are replaced by trigonometric functions, in the arguments of which γ is replaced by $\mu = \sqrt{g\kappa v^{-2} - \frac{1}{4}\kappa^2 - m^2}$. The expression Δ_{12} for an internal wave may be represented in the form

$$\Delta_{12}(m,z) = A(m) \exp \frac{\kappa(z_1 - z)}{2} \sin\left[\mu(z - z_2) + \varphi^*(m)\right], \quad (2.5)$$

where A, ϕ^* are known functions of m. It follows from (2.5) that both the nodal and the extreme points of u_n, w_n, located in the inhomogeneous layer, may be considered equidistant; in view of the smallness of ε, the zeros and extreme points of w_n are close to the extreme points and zeros of the harmonic u_n, respectively.

For $H = 2 \cdot 10^3$ m, $h_1 = 20$ m, $h_2 = 10^2$ m, $\varepsilon = 10^{-2}$, Fig. 1 shows the profiles $w_2(z)$ (a), $u_2(z)$ (b), normalized to $\theta_1 = \max w_2(z)$, $\theta_2 = u_2(0)$, respectively; the values of v are 2.2 m/sec (1a, 1b) and 1.2 m/sec (2a, 2b). The ratios $\lambda_2 h_1^{-1}$, /31 $\lambda_2 h_3^{-1}$ for curves 1a and 1b are 108.2 and 2.2, respectively, and for curves 2a, 2b, 18.1 and 0.2. It is apparent that in both cases $\lambda_2 h_1^{-1}$ is fairly large, and therefore

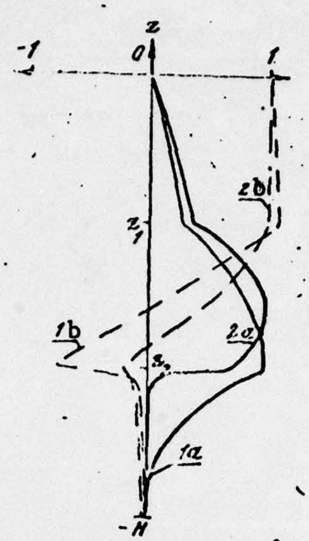


Figure 1

w_2 changes linearly in the upper layer, while u_2 is nearly constant. As v decreases, the extrema and zeros of the functions u_2, w_2 move upward, the values of $|w_2(z_i)|\theta_1^{-1}$ decrease, and $|u_2(z_i)|\theta_1^{-1}$ increase ($i = 1, 2$). The same distortion of u_2 and w_2 profiles takes place with increasing ε and layer thicknesses h_s ($s = 1, 2, 3$). As the parameters change, the harmonics $u_n(z)$ and $w_n(z)$ for $n > 2$ become similarly distorted.

It follows from the above analysis of amplitudes u_n, w_n that in expressions (1.3) and (1.4), waves with numbers $n \geqslant 2$, caused by the inhomogeneity of the liquid, are typical internal waves whose velocity amplitudes reach their extreme values in the discontinuity layer.

We introduce the quantities $A_n = \max|w_n(z)|$, $B_n = \max|u_n(z)|$. As was noted above, the values of A_1 and B_1 are close to those obtained in the case of a homogeneous liquid. For v outside a certain vicinity $v = v_1$, the wavenumber m_1 is equal to gv^{-2} to a high degree of accuracy, and the ratio $B_1A_1^{-1}$ is close to unity. This case takes place when $v < 0.2\ v_1$, and therefore, over a wide range of variation of the parameters when $v < v_2$. For internal waves, in view of the smallness of ε, we obtain from formula (2.5) the following expression for $\phi_n = B_nA_n^{-1}$ ($n \geqslant 2$):

$$\varphi_n \approx \mu_n m_n^{-1} \approx \sqrt{\frac{gK}{m_n^2 v^2} - 1}\ . \qquad (2.6)$$

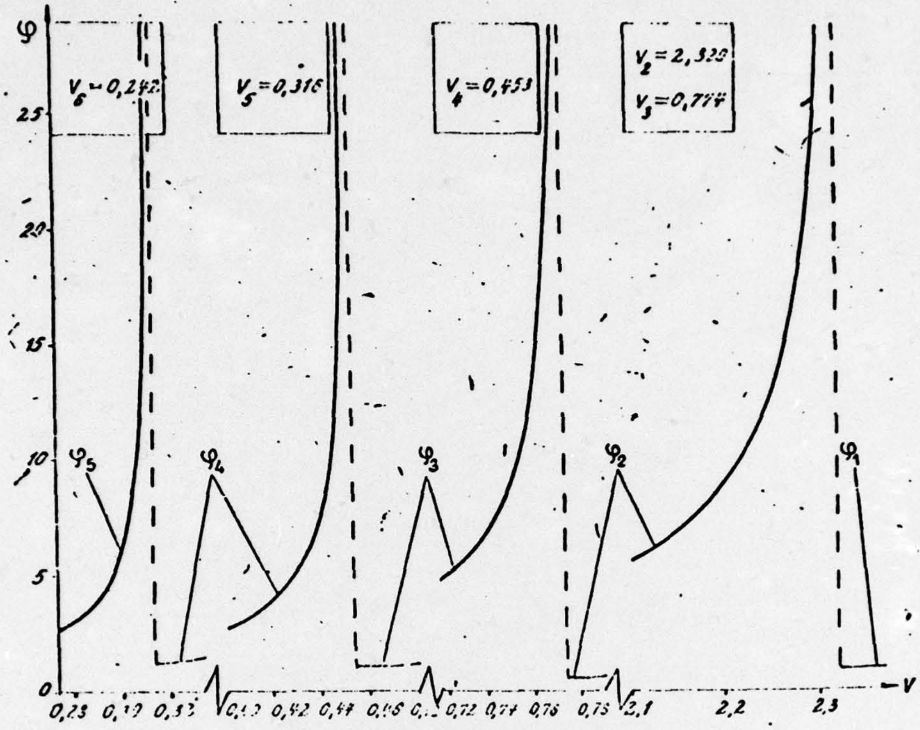


Figure 2

It can be shown that the quantity $gk/m_n^2v^2$, which decreases smoothly, tends to unity when $v \to 0$. Therefore, ϕ_n as functions of v, determined in the interval $0 \leqslant v \leqslant v_n$, increase smoothly, and $\phi_n \to 0$ when $v \to 0$. All A_n are bounded when $0 \leqslant v \leqslant v_n$, and B_n, ϕ_n, λ_n increase indefinitely if $v \to v_n - 0$. Thus, the value $v = v_n$ is a resonance value for the nth wave.

Figure 2 for $h_1 = 20$ m, $h_2 = 10^2$ m, $H = 2 \cdot 10^3$ m and $\varepsilon = 10^{-2}$ shows the curves $\phi_n = \phi_n(v)$ in the intervals (v_{n+1}, v_n) for n = 2-5. Hence it is obvious that for values of v close to but greater than v_n, the extreme value of the amplitude of the horizontal velocity for the nth wave substantially exceeds the analogous value for the vertical velocity (long waves), and for v close to but greater than v_n, these values are of the same order. When $v \to 0$, we have $A_n^2 + B_n^2 \to 0$, and therefore with decreasing velocity of displacement of the region of baric disturbances, the wave motion described by the nth wave damps out. /32

The values $v = v_n$ (n = 1, 2,...) are resonance values, and therefore particular for a linear problem. In the case of nonlinear long waves of steady type, values of v close to v_n (somewhat displaced owing to the finiteness of the wave amplitude) are

associated with solitary waves.[3,4] It is of interest to estimate the lengths of the intervals (v_{1n}, v_n) for which the solution obtained for the linear problem will be inadequate to describe internal waves in a sea. We will postulate that the distribution of baric disturbances is described by the function

$$f(x) = \begin{cases} \cos \dfrac{\pi x}{2\ell}, & |x| > \ell \\ 0, & |x| \geq \ell, \end{cases} \qquad (2.7)$$ /33

$2\ell = 2 \cdot 10^4$ m, and the condition

$$A_n + B_n < 5 \cdot 10^{-2} \qquad (a/\rho_1 g = 1). \qquad (2.8)$$

is sufficient for the applicability of linear theory. We introduce the quantitites $\Delta v_n = v_n - v_{1n}$, $t_n = \Delta v_n (v_n - v_{n+1})^{-1}$, where v_{1n} are such that for $0 \leqslant v < v_{1n}$, condition (2.8) is fulfilled. In the case discussed above (Fig. 2), $t_2 < 0.1$, $t_n < 0.08$ (n = 3, 4, 5), and the lengths of the intervals (v_{1n}, v_n) for n = 2-5 are less than 1/10 of $v_n - n_{n+1}$. Therefore, for values of v outside relatively small surroundings $v = v_n$, it may be expected that nonlinear effects will be weak. For the subsequent n, condition (2.8) should be replaced by a more exact condition: The v_n values become small, and linear theory postulates that the characteristic amplitudes of the velocities of disturbed motion are small in comparison with v.

Wave motion undamped with distance is the sum of a finite number of progressive waves of different lengths and amplitudes. A complete analysis of such nonperiodic motion cannot be carried out in the general case. Several characteristic properties of this motion can be established on the basis of numerical analysis. In the calculations, the function $f(x)$ was given in the form of (2.7).

We will analyze the contribution of individual waves to wave motion undamped with distance. To estimate the contribution of the sth wave to the velocity field, we introdude the functions $\alpha_s(z)$, $\beta_s(z)$ and the quantities c_1 and d_1, using the formulas

$$\alpha_s = 10^2(A - |w_s|)/A, \quad \beta_s = 10^2(B - |u_s|)/B, \qquad (3.1)$$

$$c_1 = 10^2 \min A / \max A, \quad d_1 = 10^2 \min B / \max B,$$

where $A = \sum_{s=2}^{N} |w_s(z)|$; $B = \sum_{s=2}^{N} |u_s(z)|$ when $v_{N+1} < v < v_N$. The functions $A(z)$ and $B(z)$ give estimates from above, independent of x, for the velocities w, u of wave motion undamped with distance; $\alpha_s(z)$, $\beta_s(z)$ give estimates of the contribution (at a distant level z) to the wave motion of all internal waves except the sth wave; c_1, d_1 make it possible to estimate the "depth" of the minima of A and B, which is important, since these quantities are zero for the sth harmonic.

Table 3 /34

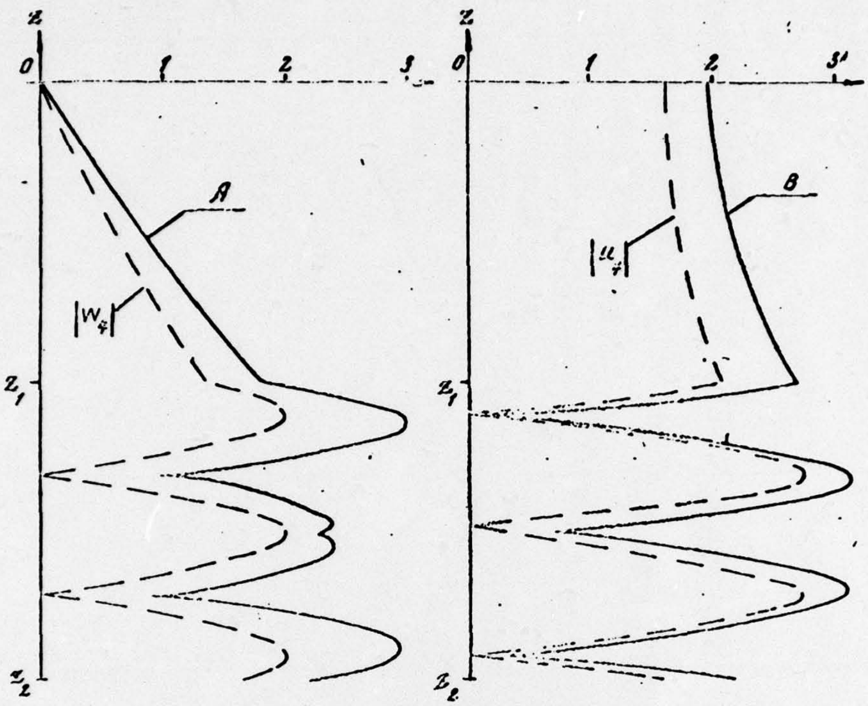
h_1	h_2	v	c_1	d_1	c_2	d_2	c_3	d_3	c_4	d_4	n
20	150	0,6	19,6	7,0	9,6	7,4	9,3	9,3	12,8	0,2	3
		0,4	36,1	22,3	27,0	22,7	26,4	26,4	32,7	11,6	4
		0,2	26,5	2,3	24,8	23,8	25,7	25,7	29,3	15,3	7
20	100	0,6	25,8	8,4	14,6	10,0	14,1	14,1	19,1	0,4	3
		0,4	10,2	2,2	5,0	3,1	5,8	4,8	6,2	1,2	4
		0,2	22,9	11,7	15,2	15,1	21,6	15,9	19,0	6,9	6
50	100	0,6	5,3	2,3	2,0	1,8	2,0	2,0	2,7	0	3
		0,4	17,1	5,1	12,1	11,3	12,0	12,0	14,6	3,3	4
		0,2	14,4	5,6	10,4	10,4	10,4	10,4	12,1	5,4	6

For a series of values of h_1, h_2 (m), v (m/sec) and $H = 2\cdot10^3$ m, $\varepsilon = 10^{-2}$, Table 3 shows c_1, d_1, $c_2 = \alpha_N(z_1)$, $d_2 = \beta_N(z_2)$, $C_3 = \alpha_N(z_2)$, $d_3 = \beta_N(z_2)$ and the quantities

$$c_4 = 10^2(\max A - \max|w_n|)/\max A, \qquad (3.2)$$
$$d_4 = 10^2(\max B - \max|u_N|)/\max B,$$

representing the perturbation of extreme values of $w_N(z)$ by waves with numbers k such that $2 \leqslant k \leqslant N - 1$. Hence, it is obvious that the largest contribution to the wave motion under the free surface of the sea is usually made by the wave of the greatest length (nth when $v_{N+1} < v < v_N$). The contribution of the remaining waves to the wave motion in these cases does not exceed 37%. Therefore, of particular importance in describing the distribution of velocity amplitudes with depth are the values of $v = v_n$, since, as the sign of $v - v_n$ changes, the dominant wave is replaced by another one, and the character of the variation of amplitudes with depth becomes qualitatively different (the number of nodal points and extrema for a given point x usually changes). For $h_1 = 20$ m, $h_2 = 150$ m, $H = 2\cdot10^3$ m, $\varepsilon = 10^{-2}$ and $v = 0.4$ m/sec (to within the factor 10^{-5} $a/\rho_1 g$), Fig. 3 shows the profiles of $A(z)$, $|w_4(z)|$ (Fig. 3a), $B(z)$, $|u_4(z)|$ (Fig. 3b) in the $z_2 \leqslant z \leqslant 0$ depth range. It is immediately apparent that the fourth wave determines the character of the variation of $A(z)$ and $B(z)$ with depth ($N = 4$).

We will consider the quantities $A_0 = \max A(z)$, $B_0 = \max B(z)$ as functions of v. /35
The presence of resonance values $v = v_n$ is responsible for the fact that the character of the variation of $A_0(v)$, $B_0(v)$ for v close to but smaller than v_n is determined by A_n, B_n. Therefore, the values of $v = v_n$ are resonance values for the total velocity field. Moreover, if v is close to v_n ($n = 1, 2,\ldots$) on the left, the



a) b)

Figure 3

horizontal wave motion is much more intense than the vertical one; for v close to v_n on the right, the extreme values of the amplitudes of the horizontal and vertical velocities are of the same order.

Let us consider the question of the dependence of the wave motion on the thicknesses of homogeneous layers. Analysis showed that when $h_3 h_2^{-1} > 2.5$, the wave motion is practically independent of h_3. To within the factor 10^{-4} $a/\rho_1 g$, for a series of values of h_3 (m) and v = 1 m/sec, h_1 = 20 m, $h_2 = 10^{-2}$, $\varepsilon = 10^{-2}$, Table 4 presents $A_1 = A(0) + |w_1(0)|$, A_0, $a_{1,2} = A(z_{1,2})$, $z = 10^2 |z_m + h_1| h_2^{-1}$, where z_m is the point where max A(z) is reached. Hence it is evident that when $h_3 > 120$ m, the values under consideration increase somewhat with increasing thickness of the lower /36
layer, but their change is small. A_1 is practically independent of h_3.

The amplitude of internal waves substantially depends on the thickness of the upper layer. When h increases, a marked decrease in the maximum possible amplitudes of internal waves takes place; this is explained by an increase in the distance of the discontinuity layer from the free surface, to which the perturbing pressures are applied. Quantities analogous to those given in Table 4 are presented in Table 5 to within the factor 10^{-5} $a/\rho_1 g$ for v = 0.6 m/sec, $h_2 = 10^2$ m, $H = 2 \cdot 10^3$ m, $\varepsilon = 10^{-2}$

Table 4

h_3	A_0	A_1	z	a_1	a_2
80	0,256	5,076	58	0,092	0,165
100	0,691	5,076	58	0,243	0,451
120	0,959	5,077	58	0,343	0,623
200	1,078	5,077	59	0,385	0,709
300	1,081	5,077	59	0,386	0,711
450	1,081	5,077	59	0,386	0,711

Table 5

h_1	A_0	A_1	a_1	a_2
20	4,936	6,629	2,997	4,035
50	2,952	6,629	2,363	2,443
10^2	0,442	6,628	0,367	0,366
$3 \cdot 10^2$	0,001	6,628	0,001	0,001

and a series of values of h_1 (m). Hence it is evident that for $h_1 = 300$ m, the amplitude of internal waves decreases by a factor of more than 10^3 in comparison with the case $h_1 = 20$ m. The surface wave motion, characterized by the quantity A_1, depends little on h_2.

It should be noted that a change in the thicknesses of homogeneous layers (at constant ε, v, h_2) causes a change in the resonance values of v_n. Cases are possible (for v close to but smaller than v_n), in which the internal waves have an amplitude /37 much greater than that of the surface wave and for a deep-lying discontinuity layer.

References

1. Fjeldstad, I. E., Interne Wellen. Geophys. Publ., Vol. 10, No. 6, 1933.

2. Krauss, V., Internal Waves (Vnutrenniye volny). Gidrometeoizdat, Leningrad, 1968.

3. Ter-Krikorov, A. M., On internal waves in an inhomogeneous liquid. "PMM," Vol. 26, No. 6, 1962.

4. Ter-Krikorov, A. M., On the theory of steady-type waves in an inhomogeneous liquid. "PMM," Vol. 29, No. 3, 1965.

5. Tareyev, B. A., On internal waves in an ocean of nonuniform density. "DAN SSSR," Vol. 149, No. 4, 1968.

6. Marchuk, G. I. and B. A. Kagan, Internal gravitational waves in a really stratified ocean. Izv. AN SSSR. Fizika atmosfery i okeana, Vol. 6, No. 4, 1970.

7. Abrashina, N. N. and L. V. Cherkesov, Internal waves due to periodic surface disturbances in a liquid with a continuously changing density. In: Morskiye gidrofizicheskiye issledovaniya, 2 (48). Sevastopol', MGI AN USSR, 1970.

8. Bukatov, A. Ye., On internal waves in a continuously stratified ocean. In: Morskiye gidrofizicheskiye issledovaniya, 6 (56). Sevastopol', MGI AN USSR, 1971.

9. Bukatov, A. Ye. and L. V. Cherkesov, Internal waves due to periodic surface disturbances. Izv. AN SSSR. Fizika atmosfery i okeana, Vol. 7, No. 8, 1971.

10. Dotsenko, S. F. and L. V. Cherkesov, Generation of internal waves by a moving region of pressures. In: Morskiye gidrofizicheskiye issledovaniya, 6 (56). Sevastopol', MGI AN USSR, 1971.

11. Dotsenko, S. F., Internal waves due to moving disturbances in a sea with an inhomogeneous upper layer. In: Morskiye gidrofizicheskiye issledovaniya, 1 (57). Sevastopol', MGI AN USSR, 1972.

12. Keller, I. B. and W. H. Munk, Internal wave wakes of a body moving in a stratified fluid. The Physics of Fluids, Vol. 13, No. 6, 1970.